**PHASE 2 REPORT**

# High-Order Homogenization Method in Diffusion Theory for Reactor Core Simulation and Design Calculation

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By

**Farzad Rahnema, PI**
Georgia Institute of Technology
Nuclear Engineering and Health Physics Programs
George W. Woodruff School of Mechanical Engineering
Atlanta, GA 30332-0405 USA
Email: farzad.rahnema@me.gatech.edu

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The following tasks were specified in the project description for Phase 2:

a) extend the homogenization method to multigroup theory;
b) extend the fine-mesh lattice code to 2-D multigroup theory;
c) extend the nodal diffusion code to 2-D multigroup theory;
d) benchmark the accuracy of the code using standard benchmark problems such the HAFAS BWR core configuration.

Goal for task (a) was met. In particular, the following were accomplished:

- A method was developed to determine the Green's function in multigroup. In this case, because of energy coupling, this function becomes a vectorial function, whereas in one-speed it is a scalar function. Substantially more effort than initially estimated was required for the numerical implementation of the solution method for the Green's function. The resulting linear systems of equations turn out to be ill-conditioned and therefore standard methods for solving them are not appropriate. After testing different approaches, a singular value decomposition method (SVD) was found to provide excellent results.

- It was shown that the perturbation expansion series for the flux, eigenvalue and the homogenized cross section converge in the multigroup case. This is new in that it has not been shown before in the literature. The benchmark configurations consisting of two types of BWR assemblies in two-group slab geometry were analyzed for various magnitudes of the perturbation in the boundary condition. The perturbation method achieves an excellent accuracy: the flux RMS error is less than 0.5% in both groups and the reference homogenized cross sections are almost reproduced.

- A summary of the work related to item a) has been accepted for presentation at

the ANS meeting in June 2002.

Regarding tasks (b) and (c), the fine-mesh lattice code as well as the nodal diffusion code had to be redeveloped for the two-group 1-D case. That is, they are not extensions of the codes used for Phase 1, which were written in JAVA. The work for Phase 1 was mostly focused on developing the general equations for the high-order cross section homogenization method and on proving the applicability and the correctness of the method for the one-speed 1-D case, for which the numerical implementation is not as complex as for the multigroup multidimensional case. When going from one to two-group theory, the form of the equations to be solved and of the expressions to be evaluated becomes more complex, due to the coupling between groups. The codes developed in Phase 2 are written in FOTRAN, which is a programming language more suitable to scientific computations and for which there are available standard libraries of functions that are needed for solving different numerical problems. The extension of the codes to 2-D has already started. It is anticipated that some preliminary results may become available at the end of Phase 2 (June 30, 2001), but not in time to be included in the current technical report.

The testing specified in item (d) for the nodal code (the two-group 1-D case) was done for five benchmark configurations typical of a BWR, from mildly to highly heterogeneous. Two of these configurations were also analyzed in one-speed 1-D in the Phase 1 study. The other three were developed during the Phase 2 period. It was concluded that for the purpose of testing the applicability and the accuracy of the homogenization method these new benchmark problems that are more realistic are needed. This was not originally included as a task for the Phase 2 period. Each assembly in the new benchmark configuration is of the GE-9 bundle design. The two-group cross sections for the assemblies used in the new benchmark problems were generated from the infinite-medium fine-mesh multigroup (45 groups) transport theory solution of a two-dimensional model of the $8 \times 8$ GE9 fuel assembly with eight burnable gadolinium absorber rods, by using the generalized geometry collision-probability code HELIOS. It is anticipated that the technical community in reactor physics and math and computations will benefit from the new benchmarks developed in this study. A full description of the new benchmark problems will be published as a technical paper this summer.

It was shown that the homogenization method provides excellent results. For all of the analyzed configurations, the node-integrated flux is within 1.16% of the assembly reference flux in all nodes for each group. There is a significant improvement from the zeroth order case (standard GET), in which the node-averaged flux has a large error (e.g., up to 8% in group 1 and up to 14% in group 2 for configuration 2). It is also shown that the reconstructed fine-mesh flux (or equivalently the power distribution) in the core approximates the reference value very well. The reference flux distribution is almost reproduced by the third order perturbation approximation.

See the summary in the phase 2 technical report for a list of publications that have and will results from the phase 2 report.